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# On Sharing Risk in Large Economies: Risk and Risk Aversion

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**Abstract:**
We study the effect that risk and risk aversion have on risk sharing in large economies with numerous agents. We show that as the economy expands, with all sectors expanding, exposure to risk is not eliminated. The risk premium does not go to zero and financial prices depend on both risk and risk-aversion, i.e., concern for risk does not disappear in the limit. Hence, while a large number of investors spreads risk, risk does not become negligible because the risk premium depends on the level of entrepreneurial activity relative to the size of the economy.

**Keywords:** Arrow-Lind Theorem, Entrepreneurial activity, Risk aversion, Risk diversification, Risk neutrality, Risk sharing, Risk spreading, Risk taking

**JEL Classification:** D81, G11

# 1 Introduction

The financial sector plays an increasingly important role in a growing and complex economy that generates more savings and investments. The financial sector influences not only the various types of risk undertaken in the economy, but also how these risks are shared among agents. One standard approach to studying the financial sector is to assume that agents are risk-averse. However, it is often assumed that as the economy grows with more agents, each agent acts less risk-averse and makes risky decisions solely on the basis of the expected rates of return, and not the underlying probability distribution of these returns.

This idea that risk-aversion becomes risk-neutrality has its origin in the literature on risk taking in the context of public investment. Namely, the Arrow-Lind Theorem, which states that risk spreading among many taxpayers renders social risk negligible (Arrow and Lind, 1970). Basically, as the number of taxpayers grows, the risk premium (corresponding to a share of the public risky project) of a risk-averse taxpayer decreases, and, in the limit, goes to zero. Hence, whether the public investment should be undertaken depends only on the expected return. Moreover, the social risk premium also goes to zero so that the government only cares about the expected return and not the risk.

The Arrow-Lind argument has been carried over to the private financial sector in an analogous way. It is argued that, with a very large number of investors, risk spreading implies that the risk premium goes to zero and thus eliminates any exposure to, and concern for risk. In other words, while the shareholders are risk-averse, there is no need to account for their risk aversion for large economies and they act as if they are risk-neutral.[1] Hence, only the expected payoff of a risky asset must be considered for trading decisions, which is then reflected in the market price.

It is the purpose of this paper to study whether risk and risk aversion have an effect on risk sharing in large economies with numerous agents. To that end, we present a model of an economy in which risky projects are undertaken

[1] See Gollier (2001, p. 317) for instance.

in the real sector and risk is shared among agents by the financial sector. Specifically, the economy is populated by both entrepreneurs and investors. Entrepreneurs are the original owners of the firms. They undertake risky projects and also issue assets. Investors share the risk of the investments with the entrepreneurs but have no entrepreneurial prospects.

After characterizing the unique equilibrium of an economy with risk sharing for a finite number of entrepreneurs and investors, we show that an expansion of the economy with all sectors expanding does not eliminate exposure to risk and that financial prices reflect risk and risk aversion. That is, in the case of non-vanishing entrepreneurial activity (i.e., the number of entrepreneurs relative to the number of investors does not approach zero, but remains a significant part of the economy), risk sharing in a large economy or in the limit economy depends on risk, risk aversion, and the level of entrepreneurial activity relative to the size of the economy. In particular, equilibrium prices are not equal to the expected returns of the assets.

We then consider the case of an expansion of the economy with vanishing entrepreneurial activity, i.e., the number of entrepreneurs becomes insignificant compared to the number of investors as the economy grows. The limiting outcomes of such an economy are identical to an economy with a finite number of risk-averse entrepreneurs and *risk-neutral* investors, and, thus, the risk premium of each agent is zero. However, the implication that a zero risk premium in the limit means that only expected payoff matters in pricing risky assets in a large economy leads to an inconsistency. Suppose that the financial markets set prices equal to the expected payoffs of the assets without any consideration for risk and risk aversion. Then, risk-averse investors receive no risk premium and do not want to engage in risk sharing. On the other hand, with the price of the asset equal to its expected value, each entrepreneur would like to sell the entire investment to the investors. This is not viable. In other words no trading is possible, which is opposite to the case of risk-neutral investors.

The paper is organized as follows. Section 2 presents the financial model and the equilibrium. Section 3 studies whether risk and risk aversion have an effect on risk sharing in large economies with numerous agents.

# 2 Model and Equilibrium

In this section, we present a model of an economy in which risky projects are undertaken in the real sector and risk is shared among agents by the financial sector. We then define and characterize the equilibrium for a finite number of agents. The limits of these equilibrium outcomes are used in the subsequent section to study whether risk and risk aversion are relevant in large economies with numerous agents.

Consider an economy with $N_E > 0$ entrepreneurs (or firms) and $N_I > 0$ investors whose objective is to maximize the expected utility of wealth. Entrepreneur $j$, the founder of firm $j$, issues equity shares that are claims on the profit $\tilde{r}_j$ generated by firm $j$. Here, $\tilde{r}_j = \theta_j + \tilde{\varepsilon}_j$ where $\theta_j$ is expected profit and $\tilde{\varepsilon}_j$ is a random shock.[2] The number of shares issued is normalized to one. Entrepreneur $j$ retains a portion of firm $j$'s shares, $\omega_{jj} \in [0, 1]$, and sells the rest in the financial market, $1 - \omega_{jj}$, to investors and other entrepreneurs in the financial market. The proceeds of the sale are invested in other risky assets and in the risk-free asset. Hence, the final wealth of entrepreneur $j$ is

$$\tilde{W}_j' = \omega_{jj}\tilde{r}_j + (1 - \omega_{jj})p_j + \sum_{k \neq j}^{N_E} (\tilde{r}_k - p_k)\omega_{jk}, \tag{1}$$

where $\omega_{jk}$ is entrepreneur $j$'s level of ownership in firm $k$ and $p_k$ is the price of a share of firm $k$. The term $(1-\omega_{jj})p_j$ is the wealth generated by entrepreneur $j$ from selling claims to the profits of firm $j$, which is diversified between the risky assets issued by firms $k \neq j$ and the risk-free asset. Specifically, entrepreneur $j$ buys $\omega_{jk}$ shares of the risky asset issued by firm $k \neq j$ at price $p_k$ with random payoff $\tilde{r}_k$, and the remaining $(1 - \omega_{jj})p_j - \sum_{k \neq j}^{N_E} p_k \omega_{jk}$ is invested in the risk-free asset with a rate of return normalized to one.

Investors do not have entrepreneurial prospects. However, they use their initial wealth to purchase shares of the risky and the risk-free asset. The

[2] We abstract from the firms' real decisions (e.g., production and output prices). See Mirman and Santugini (2012) for an analysis of monopoly behavior when the firm has access to the financial market and shares the risk with several investors.

final wealth of investor $i$ is

$$\tilde{W}_i' = W_i + \sum_{j=1}^{N_E} (\tilde{r}_j - p_j)\, z_{ij}, \tag{2}$$

where $W_i$ is initial wealth, $\tilde{r}_j - p_j$ is the random per-share return of firm $j$ stock, and $z_{ij}$ is the number of shares issued by firm $j$ that is purchased by investor $i$.

Both entrepreneurs and investors are assumed to have constant absolute risk aversion (CARA) preferences over final wealth. The random shocks have both systematic and idiosyncratic components that are normally distributed. Assumptions 2.1 and 2.2 hold for the remainder of the paper.

**Assumption 2.1.** *The coefficient of absolute risk aversion is $a_E > 0$ for any entrepreneur and $a_I > 0$ for any investor (who is not also an entrepreneur). In other words, the utility function for wealth $x$ is exponential: $u(x) = -e^{-ax}, a \in \{a_E, a_I\}$.*

**Assumption 2.2.** *For all $j, k = 1, \ldots, N_E$, $j \neq k$, $\tilde{\varepsilon}_j = \tilde{\lambda} + \tilde{\eta}_j$ where $\tilde{\lambda} \sim N(0, \sigma_\lambda^2)$ and $\tilde{\eta}_j \sim N(0, \sigma_\eta^2)$ such that $\mathbb{E}\tilde{\lambda}\tilde{\eta}_j = 0$ and $\mathbb{E}\tilde{\eta}_j\tilde{\eta}_k = 0$.*

Given Assumptions 2.1 and 2.2, there exists a strictly monotonic relationship between expected utility and the certainty equivalent. Hence, maximizing expected utility of final wealth is equivalent to maximizing the certainty equivalent. From (1), the certainty equivalent of entrepreneur $j$ is

$$CE_j\left(\{\omega_{jk}, p_k\}_{k=1}^{N_E}\right) = \mu_E\left(\{\omega_{jk}, p_k\}_{k=1}^{N_E}\right) - \pi_E\left(\{\omega_{jk}\}_{k=1}^{N_E}\right), \tag{3}$$

where the mean of final wealth is

$$\mu_E\left(\{\omega_{jk}, p_k\}_{k=1}^{N_E}\right) = \omega_{jj}\theta_j + (1 - \omega_{jj})p_j + \sum_{k \neq j}^{N_E} (\theta_k - p_k)\omega_{jk} \tag{4}$$

and the risk premium is

$$\pi_E\left(\{\omega_{jk}\}_{k=1}^{N_E}\right) = \frac{a_E}{2}\left(\sigma_\eta^2 \sum_{k=1}^{N_E} \omega_{jk}^2 + \sigma_\lambda^2 \sum_{k=1}^{N_E}\sum_{l=1}^{N_E} \omega_{jk}\omega_{jl}\right). \tag{5}$$

Similarly, from (2), the certainty equivalent of investor $i$ is

$$CE_i\left(\{z_{ij}, p_j\}_{j=1}^{N_E}\right) = \mu_I\left(\{z_{ij}, p_j\}_{j=1}^{N_E}\right) - \pi_I\left(\{z_{ij}\}_{j=1}^{N_E}\right), \tag{6}$$

where the mean of final wealth is

$$\mu_I\left(\{z_{ij}, p_j\}_{j=1}^{N_E}\right) = W_i + \sum_{j=1}^{N_E}(\theta_j - p_j)z_{ij} \tag{7}$$

and the risk premium is

$$\pi_I\left(\{z_{ij}\}_{j=1}^{N_E}\right) = \frac{a_I}{2}\left(\sigma_\eta^2 \sum_{j=1}^{N_E} z_{ij}^2 + \sigma_\lambda^2 \sum_{j=1}^{N_E}\sum_{l=1}^{N_E} z_{ij}z_{il}\right). \tag{8}$$

We now define the equilibrium. The financial sector is assumed to be perfectly competitive, i.e., prices are taken as given. Conditions 1 and 2 state the optimal policy functions for any entrepreneur and any investor, respectively. Condition 3 states that prices equate demand and supply for each asset.

**Definition 2.3.** *The tuple* $\left\{\left\{\omega_{jk}^*\left(\{p_l^*\}_{l=1}^{N_E}\right)\right\}_{k=1}^{N_E}, \left\{z_{ij}^*\left(\{p_l^*\}_{l=1}^{N_E}\right)\right\}_{i=1}^{N_I}, p_j^*\right\}_{j=1}^{N_E}$ *is an equilibrium if*

1. *Given* $\{p_k^*\}_{k=1}^{N_E}$*, for* $j = 1, \ldots, N_E$*,*

$$\left\{\omega_{jk}^*\left(\{p_l^*\}_{l=1}^{N_E}\right)\right\}_{k=1}^{N_E} = \underset{\{\omega_{jk}\}_{k=1}^{N_E}}{\arg\max}\; CE_j\left(\{\omega_{jk}, p_k^*\}_{k=1}^{N_E}\right) \tag{9}$$

2. *Given* $\left\{p_j^*\right\}_{j=1}^{N_E}$*, for* $i = 1, \ldots, N_I$*,*

$$\left\{z_{ij}^*\left(\{p_l^*\}_{l=1}^{N_E}\right)\right\}_{j=1}^{N_E} = \underset{\{z_{ij}\}_{j=1}^{N_E}}{\arg\max}\; CE_i\left(\left\{z_{ij}, p_j^*\right\}_{j=1}^{N_E}\right) \tag{10}$$

3. *Given* $\left\{\left\{\omega_{jk}^*\left(\{p_l\}_{l=1}^{N_E}\right)\right\}_{k=1}^{N_E}, \left\{z_{ij}^*\left(\{p_l\}_{l=1}^{N_E}\right)\right\}_{i=1}^{N_I}\right\}_{j=1}^{N_E}$, $\{p_j^*\}_{j=1}^{N_E}$ *clear the financial markets.*

Proposition 2.4 characterizes the unique equilibrium. For a finite number of agents, risk and risk aversion have an effect on the equilibrium outcomes (financial prices and the allocation of risk among the agents). Since the equilibrium values for risk sharing and the risk premium are the same across agents, we simplify notation. Let $\omega^* \equiv \omega^*_{jk}\left(\{p^*_l\}_{l=1}^{N_E}\right)$, $\pi^*_E \equiv \pi^*_E\left(\left\{p^*_j\right\}_{j=1}^{N_E}\right)$, $z^* \equiv z^*_{ij}\left(\{p^*_l\}_{l=1}^{N_E}\right)$, and $\pi^*_I \equiv \pi^*_I\left(\left\{p^*_j\right\}_{j=1}^{N_E}\right)$.

**Proposition 2.4.** *For $N_I, N_E < \infty$, there exists a unique equilibrium. In equilibrium,*

1. *For $j = 1, \ldots, N_E$, the price of risky asset $j$ is*

$$p^*_j = \theta_j - \frac{a_I a_E(\sigma^2_\eta + N_E\sigma^2_\lambda)}{a_I N_E + a_E N_I}. \tag{11}$$

2. *For $j = 1, \ldots, N_E$, entrepreneur $j$ holds a fraction*

$$\omega^* = \frac{a_I}{a_I N_E + a_E N_I} \tag{12}$$

*of firm $k$'s shares ($k = 1, \ldots, N_E$) so that his risk premium is*

$$\pi^*_E = \frac{a_E}{2}\left(\sigma^2_\lambda + \frac{\sigma^2_\eta}{N_E}\right)\left(\frac{a_I N_E}{a_I N_E + a_E N_I}\right)^2. \tag{13}$$

3. *For $i = 1, \ldots, N_I$, investor $i$ purchases the fraction*

$$z^* = \frac{a_E}{a_I N_E + a_E N_I} \tag{14}$$

*of firm $k$'s shares ($k = 1, \ldots, N_E$) so that his risk premium is*

$$\pi^*_I = \frac{a_I}{2}\left(\sigma^2_\lambda + \frac{\sigma^2_\eta}{N_E}\right)\left(\frac{a_E N_E}{a_I N_E + a_E N_I}\right)^2. \tag{15}$$

*Proof.* See Appendix A. □

## 3 Analysis

Having characterized the unique equilibrium of an economy with risk sharing for a finite number of entrepreneurs and investors, we next study whether risk and risk aversion have an effect on risk sharing in large economies with numerous agents as well as in the limit economy. As noted in the introduction, Arrow-Lind type of theorems suggests that with a very large number of investors, risk spreading eliminates any exposure to, and concern for risk. In other words, while the agents are risk-averse, there is no need to account for their risk aversion since they act as if they are risk-neutral. Hence, only the expected payoff of a risky asset has to be considered for trading decisions. In our model, an increase in the number of investors increases the agents' ability to spread risk among themselves, i.e., for a given number of risky assets, an increase in the number of investors reduces each agent's exposure to a particular risk.[3]

Our purpose is not to prove a theorem about the convergence of the economy but to show the effect of a limiting argument that takes account of only one aspect of the economy (the number of investors). We show that a proper expansion of the economy with all sectors expanding implies that a large number of investors does not eliminate exposure to risk and that financial prices reflect risk and risk aversion. This is true in the limit as well.

To see this, Proposition 3.1 states the limiting outcomes of the equilibrium defined in Proposition 2.4. Because there are two groups of agents, it is required to specify how different groups grow relative to each other as the economy expands, i.e., the limit of $N_E/N_I$ must be specified. Consider $\lim_{N_E,N_I\to\infty} N_E/N_I = \phi, 0 \leq \phi < \infty$, which embeds two cases. The case $0 < \phi < \infty$ refers to a growing economy with a *non-vanishing entrepreneurial activity*, i.e., entrepreneurial activity does not become insignificant as the

[3] An increase in $N_E$ has two effects. First, a higher number of entrepreneurs also increases the agents' ability to spread risk among themselves since each entrepreneur purchases shares of the risky assets issued by the other entrepreneurs. Second, a higher number of risky assets increases the agents' ability to diversify the risk in the economy. In an integrated economy, agents benefit not only from risk spreading but also from risk diversification, both of which have the effect of reducing exposure to risk. See Salanié (1997, p. 53) for instance.

economy grows. The case $\phi = 0$ refers to a growing economy with a *vanishing entrepreneurial activity*, i.e., entrepreneurial activity becomes dwarfed by the number of investors as the economy grows. From Proposition 3.1, the limiting outcomes depend on the degree of entrepreneurial activity relative to the overall economy. More entrepreneurial activity (a higher $\phi$) decreases financial prices and increases (decreases) the amount of risk born by the group of entrepreneurs (investors), which, in turn, increases (decreases) their risk premium.

**Proposition 3.1.** *Suppose that* $\lim_{N_E, N_I \to \infty} N_E/N_I = \phi, 0 \leq \phi < \infty$. *Then, from* (11) *to* (15), *in the limit,*

1. *For each asset $j$,*

   (a) *The price is*

$$\lim_{N_E, N_I \to \infty} p_j^* = \theta_j - \frac{a_I a_E \phi}{a_I \phi + a_E} \sigma_\lambda^2. \tag{16}$$

   (b) *The share of risk borne by the group of entrepreneurs is*

$$\lim_{N_E, N_I \to \infty} N_E \omega^* = \frac{a_I \phi}{a_I \phi + a_E}, \tag{17}$$

   *and the share of risk borne by the group of investors is*

$$\lim_{N_E, N_I \to \infty} N_I z^* = \frac{a_E}{a_I \phi + a_E}. \tag{18}$$

2. *The risk premium of any entrepreneur is*

$$\lim_{N_E, N_I \to \infty} \pi_E^* = \frac{a_E \sigma_\lambda^2}{2} \left( \frac{a_I \phi}{a_I \phi + a_E} \right)^2, \tag{19}$$

   *and the risk premium of any investor is*

$$\lim_{N_E, N_I \to \infty} \pi_I^* = \frac{a_I \sigma_\lambda^2}{2} \left( \frac{a_E \phi}{a_I \phi + a_E} \right)^2. \tag{20}$$

We begin with the case of a growing economy with non-vanishing entrepreneurial activity, i.e., $\phi \neq 0$. Proposition 3.2 states that neither a very large number of investors nor the limit remove the concern for risk. In other words, all risk-averse agents are exposed to risk and do not act as if they were risk-neutral.[4] Hence, Arrow-Lind type of theorems suggesting that *a large number of investors spreading risk eliminates any exposure to, and concern for risk* does not hold for an economy expanding evenly, i.e., when no group disappears and a fraction of the population has entrepreneurial prospects.

**Proposition 3.2.** *Suppose that* $\lim_{N_E, N_I \to \infty} N_E/N_I = \phi \in (0, \infty)$. *Then, from* (16) *to* (20)*,*

1. *Financial prices depend on risk and risk aversion.*

2. *The allocation of risk depends on risk aversion.*

3. *Each group bears risk, i.e.,* $\lim_{N_E, N_I \to \infty} N_E \omega^* \in (0, 1)$ *and* $\lim_{N_E, N_I \to \infty} N_I z^* \in (0, 1)$

4. *Risk premiums are nonzero, i.e.,* $\lim_{N_I, N_E \to \infty} \pi_E^* > 0$ *and* $\lim_{N_I, N_E \to \infty} \pi_I^* > 0$.

Using Proposition 3.2, we perform a comparative analysis of the effect of risk and risk aversion on equilibrium outcomes. Remark 3.3 provides a comparative analysis for financial prices. Note that, except for the idiosyncratic risk, the direction of the effects of risk and risk aversion are identical for a finite number of agents and in the limit. In the limit, the idiosyncratic risk washes away through risk diversification.

[4]Note that if everybody has entrepreneurial prospects (i.e., $N_E > 0, N_I = 0$), risk and risk aversion still matters. From (16), the limiting price $\lim_{N_E \to \infty} p_j^*|_{N_I=0} = \theta_j - a_E \sigma_\lambda^2 \neq \theta_j$ depends on risk and risk aversion.

**Remark 3.3.** *Suppose that* $\lim_{N_E,N_I\to\infty} N_E/N_I = \phi \in (0,\infty)$. *Then, from* (16), *in the limit,*

1. $$\frac{\partial\left(\lim_{N_E,N_I\to\infty} p_j^*\right)}{\partial a_E} = -\frac{a_I^2\phi^2\sigma_\lambda^2}{(a_I\phi + a_E)^2} < 0.$$

2. $$\frac{\partial\left(\lim_{N_E,N_I\to\infty} p_j^*\right)}{\partial a_I} = -\frac{a_E^2\phi\sigma_\lambda^2}{(a_I\phi + a_E)^2} < 0.$$

3. $$\frac{\partial\left(\lim_{N_E,N_I\to\infty} p_j^*\right)}{\partial \sigma_\lambda^2} = -\frac{a_I a_E\phi}{a_I\phi + a_E} < 0.$$

4. $$\frac{\partial\left(\lim_{N_E,N_I\to\infty} p_j^*\right)}{\partial \sigma_\eta^2} = 0.$$

Remark 3.4 provides a comparative analysis for the allocation of risk between the groups of entrepreneurs and investors. Note that the direction of the effects of risk and risk aversion are identical for a finite number of agents and in the limit.

**Remark 3.4.** *Suppose that* $\lim_{N_E,N_I\to\infty} N_E/N_I = \phi \in (0,\infty)$. *Then, from* (17) *and* (18), *in the limit,*

1. $$\frac{\partial\left(\lim_{N_I,N_E\to\infty} N_E\omega^*\right)}{\partial a_E} < 0, \frac{\partial\left(\lim_{N_I,N_E\to\infty} N_I z^*\right)}{\partial a_E} > 0.$$

2. $$\frac{\partial\left(\lim_{N_I,N_E\to\infty} N_E\omega^*\right)}{\partial a_I} > 0, \frac{\partial\left(\lim_{N_I,N_E\to\infty} N_I z^*\right)}{\partial a_I} < 0.$$

Having shown that the expected payoff is not sufficient to determine the equilibrium amount of risk sharing in the presence of entrepreneurs, we now turn to the restrictive case of an expansion of the economy with vanishing entrepreneurial activity, i.e., $\phi = 0$. Proposition 3.5 states that the limiting outcomes of an economy with a vanishing entrepreneurial activity are identical to an economy with a finite number of risk-averse entrepreneurs and *risk-neutral* investors.[5] Specifically, prices tend to the expected payoffs (statement 1a). The entrepreneurs relinquish all shares to the investors

[5]To derive quickly the case of an economy with risk-averse entrepreneurs and risk-neutral investors, evaluate (11) through (15) at $a_I = 0$, which yields the limiting outcomes stated in Proposition 3.5.

(statement 1b) and thus face no risk (i.e., each entrepreneur's risk premium goes to zero in statement 2). Similarly, while in aggregate the investors take on all the risk, individually they face no risk (i.e., investor's risk premium goes to zero in statement 3) as the risk is perfectly spread among themselves.

**Proposition 3.5.** *Suppose that* $\lim_{N_E,N_I\to\infty} N_E/N_I = \phi = 0$*. Then, from* (16) *to* (20)*, in the limit,*

1. *For each asset* $j$*,*

   *(a) The price is* $\lim_{N_E,N_I\to\infty} p_j^* = \theta_j$*.*

   *(b) The group of investors bear all the risk, i.e.,* $\lim_{N_E,N_I\to\infty} N_E\omega^* = 0$ *and* $\lim_{N_E,N_I\to\infty} N_I z^* = 1$*.*

2. *The risk premium of any agent is zero, i.e.,* $\lim_{N_E,N_I\to\infty} \pi_E^* = 0$*. and* $\lim_{N_E,N_I\to\infty} \pi_I^* = 0$*.*

In the limit, risk-averse investors appear to behave as if they were risk-neutral. However, risk-averse investors do not become risk-neutral in the limit, rather the gamble disappears and so does the risk.[6] Moreover, the implication that a zero risk premium in the limit means that only expected payoff matters in pricing risky assets leads to an inconsistency.

Suppose that the financial markets set prices equal to the expected payoff without any consideration for risk and risk aversion, i.e., $p_j^* = \theta_j$. Consider first an economy with risk-averse entrepreneurs and risk-neutral investors. In this case, the risk-averse entrepreneurs are always able to pass all the risk onto risk-neutral investors. Consider next a large economy with risk-averse entrepreneurs and risk-averse investors. Regardless of the number of risk-averse investors, each price-taking investor receives a zero expected return $\theta_j - p_j^* = 0$. Hence, with zero expected return, no risk-averse investor has an incentive to buy shares of a risky asset. Formally, from (6), (7), and (8)

[6] Consider $N$ agents who share a risk. Dividing a given risk more and more finely among a growing number of agents causes the risk premium to vanish not because the agents become risk-neutral but because the gamble faced by each agent disappears.

evaluated at the equilibrium prices, the certainty equivalent increases when $z_{ij} > 0$ decreases. That is, for $z_{ij} > 0$,

$$\left.\frac{\partial CE_i\left(\left\{z_{ij}, p_j^*\right\}_{j=1}^{N_E}\right)}{\partial z_{ij}}\right|_{p_j^*=\theta_j} < 0. \tag{21}$$

While there is no gain from engaging in risk sharing on the part of the investors, each entrepreneur has no incentive to hold any of the risky asset, since the price is equal to the expected payoff, and would like to push the entire investment off onto the investors. This is not viable, and, thus, no trading is possible. Hence, to reiterate,

**Remark 3.6.** *Suppose that the financial price is equal to the expected payoff of the risky asset. Then,*

1. *Risk-neutral investors take on all the risk, while*

2. *Risk-averse investors refuse to trade.*

# A Proofs

**Proof of Proposition 2.4.** From (6) and (10), the first-order conditions of investor $i$ are (in matrix form)

$$\left(\sigma_\lambda^2 \boldsymbol{J}_{N_E} + \sigma_\eta^2 \boldsymbol{I}_{N_E}\right) \boldsymbol{z}_i = \frac{1}{a_I}\boldsymbol{X}, \tag{22}$$

where $\boldsymbol{J}_{N_E}$ is an $N_E \times N_E$-dimensional matrix of 1's, $\boldsymbol{I}_{N_E}$ is an $N_E$-dimensional identity matrix, $\boldsymbol{z}_i = [z_{i1}, \ldots, z_{iN_E}]^T$, and $\boldsymbol{X} = [\theta_1 - p_1, \ldots, \theta_{N_E} - p_{N_E}]^T$. Premultiplying (22) by $\left(\sigma_\lambda^2 \boldsymbol{J}_{N_E} + \sigma_\eta^2 \boldsymbol{I}_{N_E}\right)^{-1}$ yields investor $i$'s demand for asset $j$:

$$z_{ij}^* \left(\{p_l\}_{l=1}^{N_E}\right) = \frac{\sigma_\eta^2 + (N_E - 1)\,\sigma_\lambda^2}{a_I \sigma_\eta^2 \left(\sigma_\eta^2 + N_E \sigma_\lambda^2\right)} (\theta_j - p_j) - \frac{\sigma_\lambda^2}{a_I \sigma_\eta^2 \left(\sigma_\eta^2 + N_E \sigma_\lambda^2\right)} \sum_{l \neq j}^{N_E} (\theta_l - p_l), \tag{23}$$

$j = 1, \ldots, N_E$.

From (3) and (9), the first-order conditions of entrepreneur $j$ are (in matrix form)

$$\left(\sigma_\lambda^2 \boldsymbol{J}_{N_E} + \sigma_\eta^2 \boldsymbol{I}_{N_E}\right) \boldsymbol{\omega}_j = \frac{1}{a_E}\boldsymbol{X}, \tag{24}$$

where $\boldsymbol{\omega}_j = [\omega_{j1}, \ldots, \omega_{jN_E}]^T$. Premultiplying (24) by $\left(\sigma_\lambda^2 \boldsymbol{J}_{N_E} + \sigma_\eta^2 \boldsymbol{I}_{N_E}\right)^{-1}$ yields entrepreneur $j$'s demand for asset $k$:

$$\omega_{jk}^* \left(\{p_l\}_{l=1}^{N_E}\right) = \frac{\sigma_\eta^2 + (N_E - 1)\,\sigma_\lambda^2}{a_E \sigma_\eta^2 \left(\sigma_\eta^2 + N_E \sigma_\lambda^2\right)} (\theta_k - p_k) - \frac{\sigma_\lambda^2}{a_E \sigma_\eta^2 \left(\sigma_\eta^2 + N_E \sigma_\lambda^2\right)} \sum_{l \neq k}^{N_E} (\theta_l - p_l)\,, \tag{25}$$

$k = 1, \ldots, N_E$.

The financial price for asset $j$ satisfies the market clearing condition

$$1 - \omega_{jj}^* \left(\{p_l^*\}_{l=1}^{N_E}\right) = \sum_{i=1}^{N_I} z_{ij}^* \left(\{p_l^*\}_{l=1}^{N_E}\right) + \sum_{k \neq j}^{N_E} \omega_{kj}^* \left(\{p_l^*\}_{l=1}^{N_E}\right), \tag{26}$$

$j = 1, \ldots, N_E$. Plugging (23) and (25) into (26) and solving for equilibrium prices yields (11). Plugging (11) into (23) and (25) yields (12) and (14), respectively. Plugging (12) and (14) into (5) and (8) yields (13) and (15), respectively.

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